Using eigenvector angles to statistically evaluate the influences of connectivity structure on correlation structure

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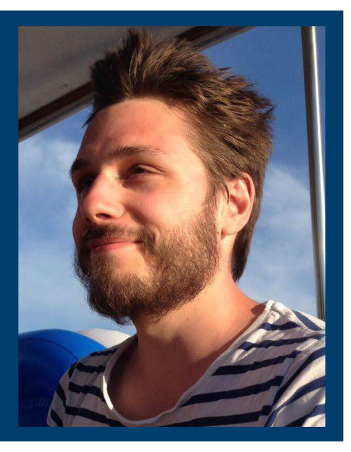
Code & Tutorial

gin.g-node.org/INM-6/eigenangles



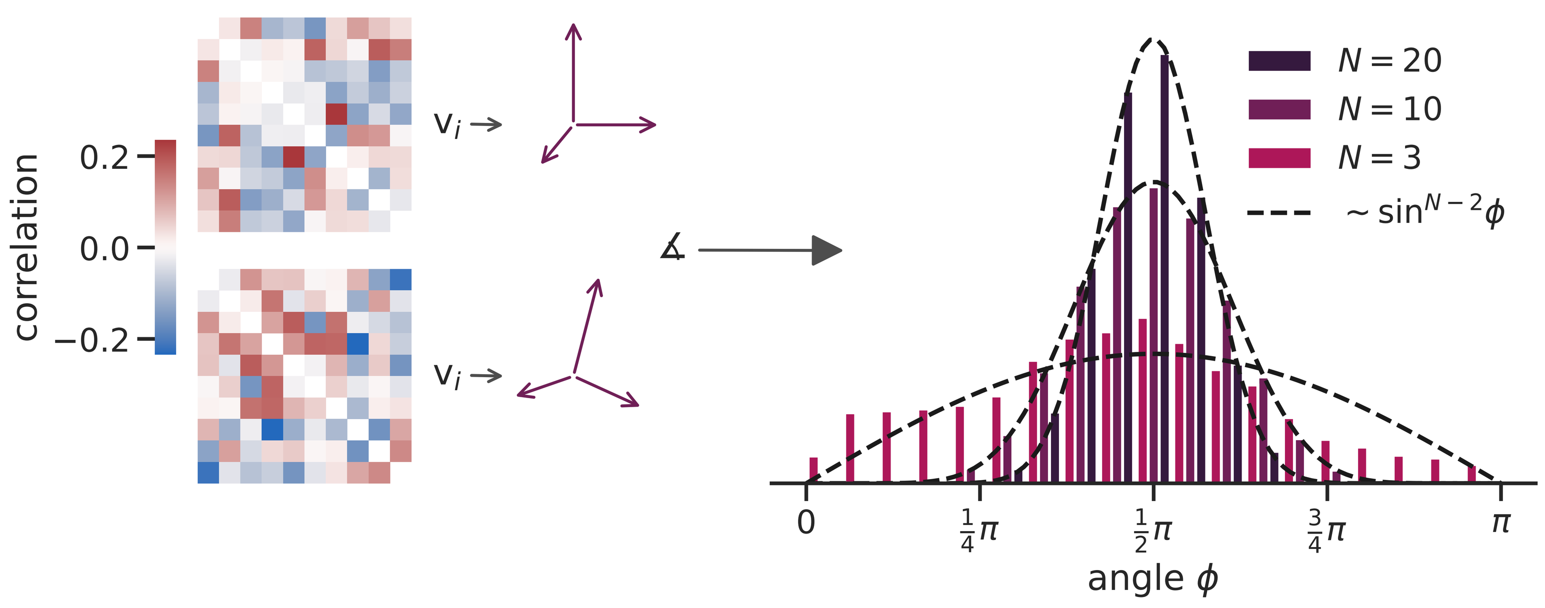
The eigenangle test quantifies the similarity of correlation matrices

Problem

How to compare pairwise measures (e.g. spiking correlation), represented by a matrix, between networks while preserving their higher-order structure?

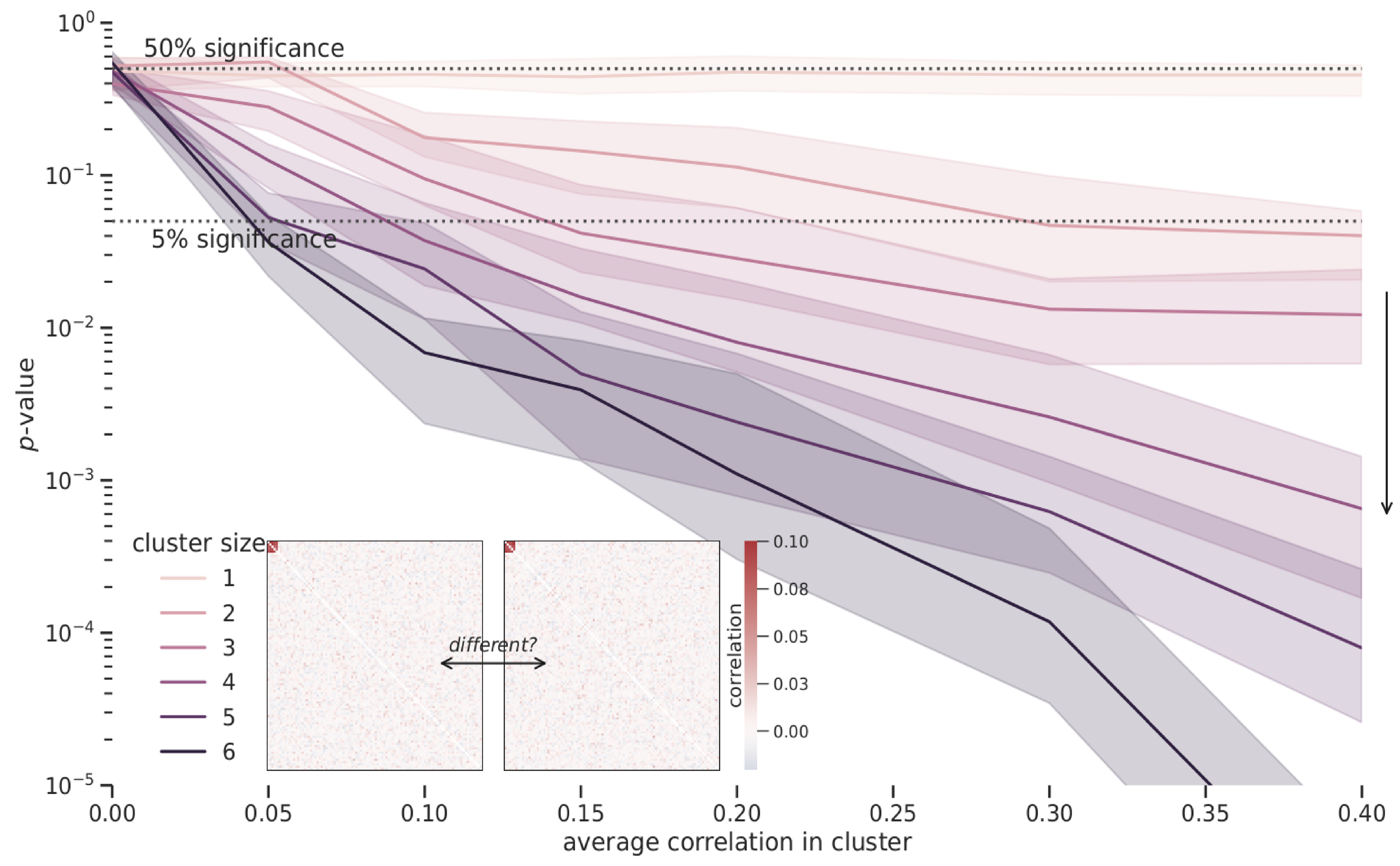
Idea

- When two network have the same subgroup of correlated neurons:
  → The corresponding eigenvectors of the correlation matrix are similar.
  → The angle between the pair of eigenvectors ("eigenangle") is small.
- When the symmetric correlation matrices are independent:
  → The eigenangles follow the random angle distribution [1].



Approach

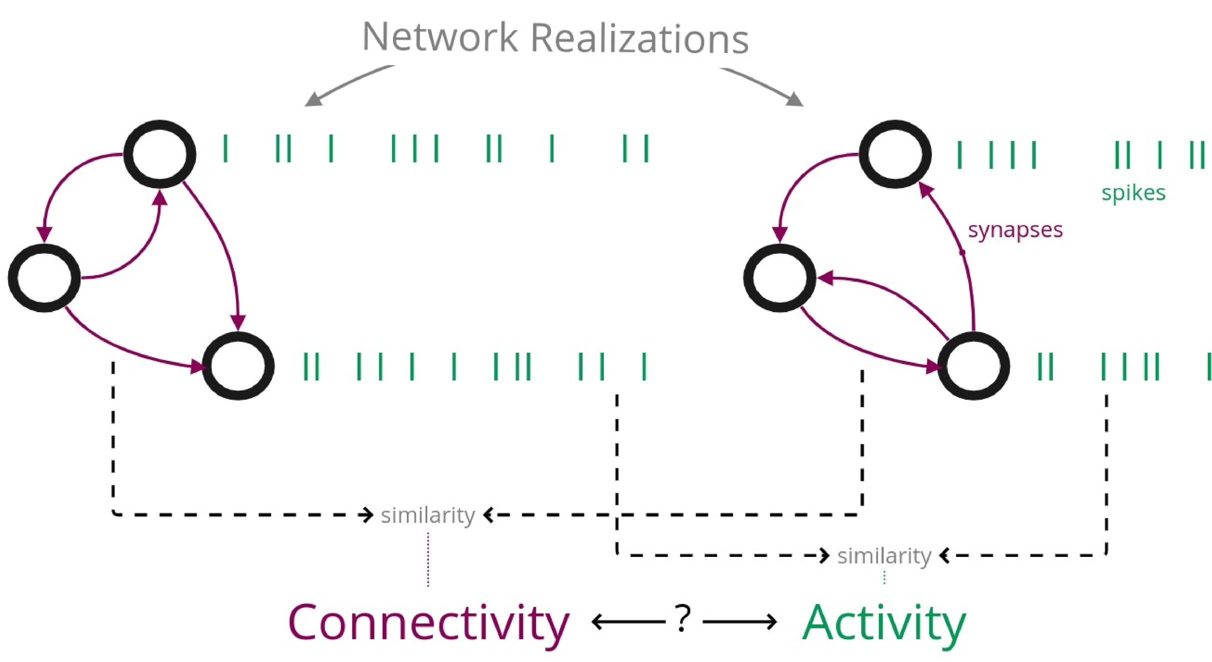
- Similarity score: average eigenangle smallness weighted by the eigenvalues.
- Null hypothesis: the two matrices are independent random matrices.
- We derive a corresponding null distribution associating each measured similarity score with a p-value.
- small p-value → reject null hypothesis → indication of similarity
- Calibrations of the "eigenangle test" shows sensitivity to correlation clusters (*see Fig.*) and ability to differentiate between pairwise and higher-order correlations.



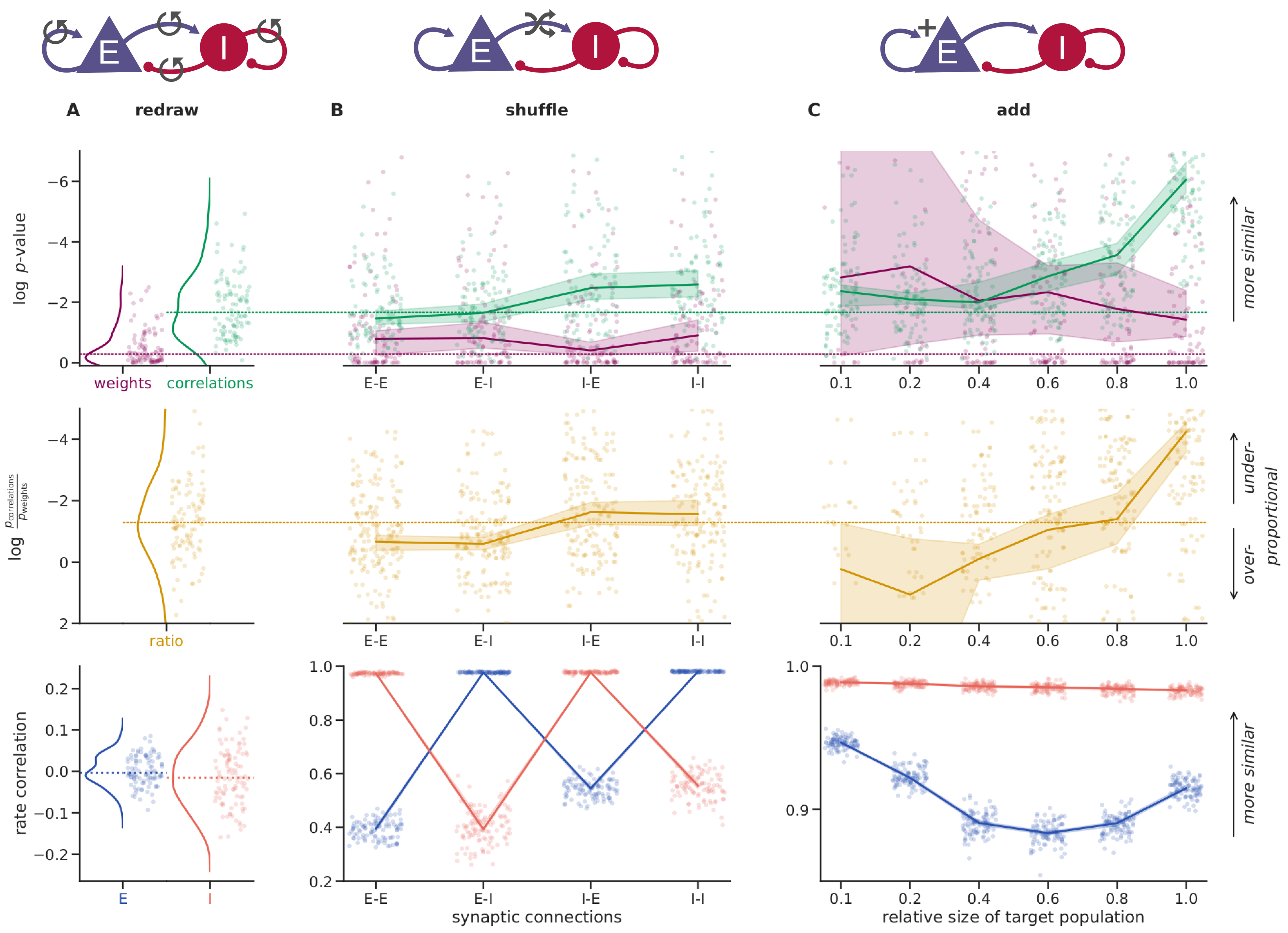
Measuring network rewiring effects on the correlation and connectivity structure

Application

- The eigenangle test can be adapted for asymmetric connectivity matrices when their random eigenvalue distribution is known.
- We use a balanced E-I network according to [2].
- We compare two network realizations on the level of their connectivity and their activity correlation using the same eigenangle measure.



- Changing the connectivity with three different rewiring protocols (*Fig. columns*), inspired by [3], we measure the resulting changes to the correlation and connectivity structure via p-values (*Fig. 1st row*), each dot is a comparison of two network realizations.
- Their p-value ratio indicates whether the effect on the correlation structure is proportional to the amount of connectivity rewiring (*Fig. 2nd row*).
- Replicating [3], we also measure the correlation of the excitatory (*blue*) and inhibitory (*red*) firing rate vectors (*Fig. 3rd row*).



Results

- The exact effects of synaptic rewiring strongly depend on the type of network model and its parameters.
- Correlation structures can remain similar even if redrawing all synaptic connections, when there are persistent population fluctuations (*A*).
- The correlation structure is mostly determined by the excitatory synapses (*B*), even when correcting for their larger number w.r.t. inhibitory synapses.
- Inhibitory synapses have more influence on the firing rates, but excitatory synapses compensate by their larger number.
- The addition of few targeted synapses changes the correlation structure more than many distributes synapses (*C*).

The eigenangle test for correlation matrices is integrated into the Python validation test library NetworkUnit [4] as of release v0.2 (*https://friends.python-elephant.org*)

References

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Acknowledgements

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